Class-XII

Miscellaneous exercise.

Q.17. $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$ is equal to

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(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $-\frac{3\pi}{4}$

Soln: $\because \tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$

$=\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{\frac{x}{y}-1}{\frac{x}{y}+1}\right)$

$=\tan^{-1}\left(\frac{x}{y}\right)-\left\{\tan^{-1}\frac{x}{y}-\tan^{-1}1\right\}$ $\left[\because \tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$

$=\cancel{\tan^{-1}\left(\frac{x}{y}\right)}-\cancel{\tan^{-1}\frac{x}{y}}+\tan^{-1}1.$

$=\tan^{-1}1=\tan^{-1}\left(\tan\frac{\pi}{4}\right)=\frac{\pi}{4}$

Hence, the correct option is (c) $\frac{\pi}{4}$ Ans.

Most Important question

Q.1 Find the value of $\tan^{-1}\left[\tan\frac{5\pi}{6}\right]+\cos^{-1}\left(\cos\frac{13\pi}{6}\right]$

Soln: $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

$=\tan^{-1}\left[\tan\left(\frac{\pi}{2}+\frac{\pi}{3}\right)\right]+\cos^{-1}\left[\cos\left(2\pi+\frac{\pi}{6}\right)\right]$

$=\tan^{-1}\left[-\tan\frac{\pi}{3}\right]+\cos^{-1}\left(\cos\frac{\pi}{6}\right)$

$\left[\because \tan\left(\frac{\pi}{2}+\theta\right)=-\tan\theta \text{ and } \cos(2\pi+\theta)=\cos\theta\right]$

$=\tan^{-1}\left(-\tan\left(\frac{\pi}{2}-\frac{\pi}{3}\right)+\cos^{-1}\left[\cos\frac{\pi}{6}\right]\right.$ $\left[\because \tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta\right]$

$=\tan^{-1}\left[-\tan\frac{\pi}{6}\right]+\cos^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right]$

$=\tan^{-1}\left[\tan\left(-\frac{\pi}{6}\right)\right]+\frac{\pi}{6}$ $\left[\because \tan(-\theta)=-\tan\theta\right]$

$=-\cancel{\frac{\pi}{6}}+\cancel{\frac{\pi}{6}}=0.$

Q.2 Evaluate $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$

Soln: $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]=\cos\left[\pi-\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$

$\left[\because \cos^{-1}(-x)=\pi-\cos^{-1}x\right]$

$=\cos\left[\pi-\cancel{\frac{\pi}{6}}+\cancel{\frac{\pi}{6}}\right]=\cos\pi=-1$

Ans

Class XII Important Question

Q.3 Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$

Soln: $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$

$$=\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left(-\sin\frac{\pi}{2}\right)$$

$$=-\tan^{-1}\frac{1}{\sqrt{3}}+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)-\tan^{-1}\left(\sin\frac{\pi}{2}\right) \quad [\because \sin(-\theta)=-\sin\theta]$$

$$=-\tan^{-1}\left(\tan\frac{\pi}{6}\right)+\cot^{-1}\left(\cot\frac{\pi}{3}\right)-\tan^{-1}(1) \quad [\because \tan^{-1}(-x)=-\tan^{-1}x, \ \sin\frac{\pi}{2}=1]$$

$$=-\frac{\pi}{6}+\frac{\pi}{3}-\tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$=-\frac{\pi}{6}+\frac{\pi}{3}-\frac{\pi}{4}=\frac{-2\pi+4\pi-3\pi}{12}=-\frac{\pi}{12} \text{ Ans.}$$

Q.4. Find the real solution of the equation

$$\tan^{-1}\left(\sqrt{x(x+1)}\right)+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$$

Soln: $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$

$$\Rightarrow \tan^{-1}\sqrt{x^2+x}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$$

This eqn holds, if

$x^2+x\geq 0$ and $0\leq x^2+x+1\leq 1$

Now, $x^2+x\geq 0$ and $0\leq x^2+x+1\leq 1$

$\Rightarrow x^2+x\geq 0$ and $x^2+x+1\leq 1$ $[\because x^2+x+1>0 \ \forall x]$

$\Rightarrow x^2+x\geq 0$ and $x^2+x\leq 0$

$\Rightarrow x^2+x=0 \Rightarrow x(x+1)=0$

$\Rightarrow x=0$ or, $x+1=0$

$\Rightarrow x=0$ or, $x=-1$

Hence, $x=0,-1$ Ans.

Class-XII Inverse Trigonometric Functions

Miscellaneous Exercise.

Q.14. $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x, (x>0)$

Soln: It is given that

$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x \Rightarrow 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1}x$

$\Rightarrow \tan^{-1}\left[\frac{2\left(\frac{1-x}{1+x}\right)}{1-\left(\frac{1-x}{1+x}\right)^2}\right] = \tan^{-1}x \quad \left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$\Rightarrow \tan^{-1}\left[\frac{\frac{2(1-x)}{(1+x)}}{\frac{(1+x)^2-(1-x)^2}{(1+x)^2}}\right] = \tan^{-1}x$

$\Rightarrow \tan^{-1}\left[\frac{2(1-x)}{\cancel{(1+x)}} \times \frac{(1+x)^{\cancel{2}}}{(1+x)^2-(1-x)^2}\right] = \tan^{-1}x$

$\Rightarrow \tan^{-1}\left[\frac{2(1-x)(1+x)}{\cancel{1}+2x+\cancel{x^2}-\cancel{1}+2x-\cancel{x^2}}\right] = \tan^{-1}x$

$\Rightarrow \tan^{-1}\left[\frac{2(1-x^2)}{4x}\right] = \tan^{-1}x$

$\Rightarrow \cancel{\tan^{-1}}\left[\frac{1-x^2}{2x}\right] = \cancel{\tan^{-1}}x$

$\Rightarrow \frac{1-x^2}{2x} = x \Rightarrow 1-x^2 = 2x^2 \Rightarrow 1 = 2x^2+x^2 \Rightarrow 1 = 3x^2$

$\Rightarrow 3x^2 = 1 \Rightarrow x^2 = \frac{1}{3} \Rightarrow x = \pm\frac{1}{\sqrt{3}}$

$[\because x>0$ given, then $x \neq -\frac{1}{\sqrt{3}}$

$\therefore x = \frac{1}{\sqrt{3}}$

Q.15. $\sin(\tan^{-1}x), |x|<1$, is equal to

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$ ✓

Soln: $\sin(\tan^{-1}x) = \sin\left[\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right] \quad \left[\because \tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}}\right]$

$= \frac{x}{\sqrt{1+x^2}}$

Hence, the correct option is (d). Ans

Class-XII Miscellaneous Exercise

Q.16. $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, then x is equal to

(a) $0, \frac{1}{2}$ (b) $1, \frac{1}{2}$ (c) 2020 (d) $\frac{1}{2}$

Soln: It is given that $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

Let $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$, then

$\sin^{-1}(1-\sin\theta) - 2\theta = \frac{\pi}{2} \Rightarrow \sin^{-1}(1-\sin\theta) = \frac{\pi}{2} + 2\theta$

$\Rightarrow 1 - \sin\theta = \sin(\frac{\pi}{2} + 2\theta) \Rightarrow 1 - \sin\theta = \cos 2\theta$

$\Rightarrow \cancel{1} - \sin\theta = \cancel{1} - 2\sin^2\theta$ $[\because \sin(\frac{\pi}{2}+\theta) = \cos\theta]$

$\Rightarrow -\sin\theta = -2\sin^2\theta$ $[\because \cos 2\theta = 1 - 2\sin^2\theta]$

$\Rightarrow \sin\theta = 2\sin^2\theta \Rightarrow \sin\theta(2\sin\theta - 1) = 0$

Either $\sin\theta = 0$ or $2\sin\theta - 1 = 0$

$\Rightarrow x = 0$ or $2x - 1 = 0$ $[\because \sin\theta = x]$

$\Rightarrow x = 0$ or $2x = 1$

$\Rightarrow x = 0$ or $x = \frac{1}{2}$

But $x = \frac{1}{2}$ does not satisfy the given equation. So $x = 0$

The option (c) 0 is correct.

Check.

Put $x = 0$ in the given eqn.

$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1-0) - 2\sin^{-1}0 = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1) - 2\sin^{-1}(0) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(\sin\frac{\pi}{2}) - 2\times 0 = \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{2} - 0 = \frac{\pi}{2}$

Put $x = \frac{1}{2}$ in the given eqn

$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

$\sin^{-1}(1-\frac{1}{2}) - 2\sin^{-1}\frac{1}{2} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{2} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(\sin\frac{\pi}{6}) - 2\sin^{-1}(\sin\frac{\pi}{6}) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(\sin\frac{\pi}{6})$

$\Rightarrow \frac{\pi}{6} - 2\times\frac{\pi}{6} = \frac{\pi}{2}$

$\Rightarrow \frac{\pi - 2\pi}{6} = \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{6} \neq \frac{\pi}{2}$

So, $x = \frac{1}{2}$ is not possible.